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A realistic unified gauge coupling from the micro-landscape of orbifold GUTs

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Abstract

We consider 5-dimensional supersymmetric field theories where supersymmetry is broken by the Scherk–Schwarz mechanism (or, equivalently, by the F-term VEV of the radion). In such models, the radion effective potential is calculable in terms of the 5d gauge coupling, the UV cutoff of the 5d field theory, and the field content. We provide simple, explicit formulae for the leading part of the two-loop effective potential. Our analysis applies in particular to 5d orbifold GUTs motivated by heterotic orbifold models. We focus on potentially realistic models of this type and make the additional assumption that the UV cutoff scale is identical with the strong-coupling scale of the 5d gauge theory. Given our stabilization mechanism, the 5d radius is now fixed in terms of the 5d gauge coupling and the field content of the model. This implies a prediction for the effective 4d gauge coupling only in terms of the field content of the model. Given the 'micro-landscape' provided by the different possible distributions of Standard Model fields between bulk and branes, we find a subset of models with a realistic unified gauge coupling. We also discuss two possibilities for the 'uplifting' of our SUSY-breaking AdS vacua: One is based on the possible presence of a weak warping, the other appeals to F-terms in an extra brane-localized SUSY-breaking sector.

1. Introduction

Supersymmetric grand unification at a scale of $\sim 10^{16}$ GeV is one of the best motivated proposals for physics beyond the standard model [1,2]. It fits rather naturally into the framework of heterotic string theory, where a large class of potentially realistic constructions with gauge cou-

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pling unification can be obtained in orbifold model building [3] (see [4] for a recent review). One of the possibilities for overcoming the string-scale GUT-scale problem [5], which generically affects these scenarios, is the compactification on anisotropic orbifolds [6–9], where at least one of the compactification radii is much larger than the string length scale. Such models have a useful effective description in terms of higher-dimensional field theories valid at energies between the string scale and the compactification scale. They are also known as orbifold GUTs which provide, independently of a possible string-theoretic UV completion, some of the simplest realistic models of grand unification [10–12].

It is therefore essential to understand possible stabilization mechanisms for the largest compact dimensions at a quantitative level. Here, we focus on the simple case of 5d supersymmetric gauge theories on S^1/Z_2 or $S^1/(Z_2 \times Z_2')$ with hypermultiplets in the bulk and chiral matter localized at the boundaries. In orbifold GUTs of this type, the 4d gauge coupling is given by

$$g_4^2 = \frac{g^2}{2\pi R}, \tag{1}$$

where g is the 5d gauge coupling and R is the compactification radius.[1] It will be instructive to rewrite this relation in terms of the parameters

$$\frac{g_4^2 N}{16\pi^2} \quad \text{and} \quad \frac{g^2 N}{24\pi^3} \equiv \frac{1}{M}, \tag{2}$$

which govern the perturbative series of an $SU(N)$ gauge theory in 4 and in 5 dimensions [13]. Note that in the 5d case, loop corrections are proportional to positive powers of (Λ/M), where Λ is the cutoff scale. Hence M can also be viewed as the 'fundamental scale' or 'strong-coupling' scale of the 5d theory: It is the highest scale to which the cutoff can be raised in perturbative effective field theory.

In terms of the proper expansion parameters given in Eq. (2), the expression for the 4d gauge coupling, Eq. (1), takes the form

$$\frac{g_4^2 N}{16\pi^2} = \frac{3}{4}\frac{1}{MR}. \tag{3}$$

This formulation shows a rather precise connection between 4d and 5d perturbativity: A strongly coupled 4d effective theory emerges when the compactification scale is raised to the 5d strong-coupling scale M. Hence, when it comes to numbers, it is more convenient to think of $1/M$ rather than of g^2 as of the parameter defining the 5d gauge theory.

For the phenomenological value $\alpha_{\text{GUT}} \simeq 1/25$ and an $SU(5)$ gauge group, the l.h. side of Eq. (3) takes a value $\simeq 1/60$. Thus, we need a corresponding hierarchy between $1/R$ and M. We will see that such a mild hierarchy[2] is relatively easy to achieve: Given the discrete set of models provided by different distributions of matter fields between bulk and branes, one finds many situations where the Casimir energy stabilizes the radius at the desired scale.

Before describing our specific results in more detail, we recall the generic situation: It is well known that the compactification radius R (i.e. the radion field in 4d language) is a modulus at

[1] We view the 4d theory as resulting from a projection applied to the 4d spectrum of an S^1-compactified 5d theory. Hence the S^1 volume $2\pi R$ rather than the orbifold volume πR or $\pi R/2$ appears.

[2] Independently of the specific value of α_{GUT}, the requirement of small but not extremely small 4d gauge couplings after compactification is common to many models with extra dimensions. Hence our analysis is relevant not only for higher-dimensional GUT models, but also for models with intermediate or TeV scale extra dimensions.

tree-level. Loop corrections lift the flatness of its effective potential [14]. If all bulk fields are massless, this 'Casimir energy' is $\propto 1/R^4$ at one-loop order on dimensional grounds. Radius stabilization requires a more complicated functional form of the effective potential and hence the presence of another mass scale. This scale can be provided, for example, by warping [15], by massive bulk matter or by brane-localized kinetic terms for bulk fields [16]. These and other mechanisms for radion stabilization have also been discussed by many authors in the context of models with spontaneously broken supersymmetry (see e.g. [17–21]). In the present, orbifold-GUT motivated context, Casimir stabilization has recently been analyzed in 6d, using brane-localized soft terms and FI-terms to provide the required mass scale [22].

We base our analysis on the observation that Casimir stabilization can occur even in the minimal realistic setting of a 5d gauge theory [23]. If it does, one has more predictive power than in many of the more elaborate constructions mentioned above. The idea is simply to use the two-loop effective potential, which is of the form $1/R^4 + g^2/R^5$ for an S^1 compactification. For appropriate numerical coefficients, a perturbatively controlled minimum at relatively large R can arise.[3] For an S^1/Z_2 or $S^1/(Z_2 \times Z_2')$ orbifold, the two-loop contribution is enhanced by a factor $\ln(\Lambda R)$, where Λ is the UV cutoff scale of the 5d field theory. This enhancement originates in the UV divergence of brane localized gauge-kinetic terms. For $\Lambda \gg 1/R$, the logarithm is large and predictivity is maintained even in the presence of unknown tree-level brane operators (as long as they are not unnaturally large). In [23], these ideas were worked out in the case of S^1 for supersymmetric and non-supersymmetric models and in the case of S^1/Z_2, but without supersymmetry or gauge symmetry breaking by orbifolding.

If we make the assumption that the cutoff or UV-completion scale Λ takes its highest possible value, $\Lambda \simeq M$, the potential takes the form

$$V(R) \sim \frac{1}{R^4} + \frac{g^2}{R^5}\ln(MR) \sim \frac{1}{R^4}\left(1 + \frac{\ln(MR)}{MR}\right). \tag{4}$$

The numerical coefficients of the two competing terms, which have been suppressed for brevity, can have different signs and values. Their ratio, which depends only on the field content of the model, determines the position of the minimum. For appropriate field content, the minimum is at $R \gg M^{-1}$, rendering our analysis self-consistent.

It is clear that the value of R at the minimum is proportional to g^2 or, equivalently, to $1/M$. The proportionality factor is calculable in terms of the field content of the model. Hence Eq. (1) provides a prediction of the 4d gauge coupling, even though we cannot determine the values of M and R independently. Of course, there are good reasons to believe that R^{-1} is of the order of $M_{\mathrm{GUT}} \sim 10^{16}$ GeV, which would require the 5d model to be characterized by $M \simeq 45 M_{\mathrm{GUT}}$. However, we emphasize again that the overall uncertainties of these scales do not affect our prediction of g_4. This prediction is based only on the quantity MR, which is calculable in terms of the gauge group, symmetry breaking pattern and matter content of the 5d orbifold model.

In the present paper, we analyze two-loop Casimir stabilization in the potentially realistic case of supersymmetric S^1/Z_2 or $S^1/(Z_2 \times Z_2')$ orbifolds with gauge symmetry breaking by boundary conditions. Although both supersymmetry (with Scherk–Schwarz breaking [26]) and gauge symmetry breaking have a significant effect on the Casimir energy, the potential can be derived essentially without new loop calculations. This is achieved using simple arguments based on the $\mathcal{N} = 2$ SUSY case and elementary group theory.

[3] Note that different two-loop Casimir stabilization mechanisms have been discussed in the context of 6d $\lambda\phi^3$ theory [24] and 5d $\lambda\phi^4$ theory [25].

It is essential for our analysis that SUSY breaking is dominated by the F-term VEV of the radion superfield T, which contains R as the real part of its scalar component. This situation corresponds to Scherk–Schwarz breaking in the rigid SUSY approximation [27,28]. The SUSY breaking scale is proportional to the (dimensionless) Scherk–Schwarz twist parameter or, equivalently, F_T. This is a small number, the square of which enters the radion potential as an overall prefactor. Hence, the precise scale of SUSY breaking is irrelevant for the position of the minimum.

We apply our general results to some $SU(5)$ orbifold GUT models. We find that the possibility of Casimir stabilization depends crucially on the distribution of matter fields between bulk and branes. (This freedom corresponds to what we previously called the 'field content' of the model.) As described in more detail above, the stabilization radius in units of g^2 directly determines the value of the 4d gauge coupling at the compactification scale. Since we aim at potentially realistic models, we need to ensure that this value is consistent with the phenomenological value of the unified gauge coupling. Within the 'micro-landscape' arising from the possible localization of matter fields, several examples with a realistic gauge coupling can be found.

Our paper is organized as follows. In Section 2, we review the most important results of Ref. [23] and explain our strategy for determining the unified gauge coupling in more detail. In Section 3, we derive general formulae for the radion potential for supersymmetric gauge theories on S^1/Z_2 with charged hypermultiplets in the bulk. This analysis is extended to situations with broken gauge symmetry in Section 4 and to $S^1/(Z_2 \times Z_2')$ compactifications with gauge symmetry broken at one of the branes in Section 5. We apply these results to simple realistic GUT models in Section 6 and identify models in which a realistic 4d gauge coupling is dynamically realized. Since the Casimir energy at the stabilization point is negative, some form of 'uplifting' is required. This issue is addressed in Section 7. The conclusions, given in Section 8, are followed by Appendix A, where we describe part of the underlying component field calculation in more detail. Although our results, as emphasized above, can be obtained without any explicit new loop calculations, we find this useful in view of a disagreement with some of the component-field results of [29].

2. Perturbatively controlled radius stabilization by Casimir energy

Let us first consider 5d gravity (with vanishing cosmological constant) and pure gauge theory, compactified on S^1:

$$\int d^4x \int_{-\pi R}^{\pi R} dy \sqrt{-\det(g_{MN})}\left(\frac{1}{2}M_{P,5}^3\mathcal{R}_5 + \frac{1}{2g^2}\operatorname{tr}\left(F_{MN}F^{MN}\right)\right). \tag{5}$$

The parameters $M_{P,5}$ and g are defined in the uncompactified 5d effective theory at zero momentum. We view this as an effective quantum field theory in which the cutoff can be taken as high as the strong-interaction scale. Compactifying this theory on S^1 then corresponds to an IR modification which should not introduce any new infinities and hence no cutoff dependence. Thus, the 4d effective potential for R (i.e. the Casimir energy) can only depend on g and $M_{P,5}$. Gauge loop effects are suppressed by powers of g^2/R, while gravitational loop effects are suppressed by powers of $1/(M_{P,5}R)^3$. Hence, the latter are subdominant in situations where $1/g^2 \lesssim M_{P,5}(M_{P,5}R)^2$. Neglecting gravitational interactions, we have

$$V(R)=\frac{1}{R^4}\left(c^{(1)}+c^{(2)}\frac{g^2}{R}+c^{(3)}\left(\frac{g^2}{R}\right)^2+\cdots\right), \tag{6}$$

where $c^{(i)}$ is the coefficient of the ith loop order contribution.[4] As explained above, the $c^{(i)}$s are cutoff-independent calculable numbers.

Given that $c^{(1)}<0$ and $c^{(2)}>0$, one finds a minimum at

$$R_{\min}=-\frac{5}{4}\frac{c^{(2)}}{c^{(1)}}g^2 \tag{7}$$

at two-loop order.[5] Unfortunately, if all $c^{(i)}$s are $\mathcal{O}(1)$, the loop expansion parameter g^2/R is also $\mathcal{O}(1)$ in the vicinity of $R\sim R_{\min}$. Hence, this minimum is not perturbatively controlled.

However, in special cases where $c^{(1)}$ is negative and $\mathcal{O}(1)$ while $c^{(2)}$ is large and positive, the loop expansion factor g^2/R is small for R close to $R_{\min}$. Higher-loop contributions to the effective potential are then suppressed by powers of g^2/R. Perturbative control is guaranteed if the possible growth of $c^{(i)}$ (for $i>2$) with i is overwhelmed by the increasing powers of g^2/R. This is indeed easily realized in simple models [23].

In essence, this strategy of finding a perturbatively controlled minimum by tuning the coefficients $c^{(1)}$ and $c^{(2)}$ via the field content also applies to orbifold compactifications. However, the power-law behavior of the effective potential, Eq. (6), is in general modified. The reason is the presence of brane-localized operators in the effective action. The prime example is a brane-localized contribution to the gauge-kinetic term. Such terms were first studied in the context of orbifold GUTs, where they can be employed to achieve gauge coupling unification by a modified logarithmic running above the compactification scale [11,30–32]. These terms are logarithmically UV-sensitive. Unless the UV-completion of our model is known, the coefficients at the cutoff scale Λ are free parameters. For reasons of naturalness, we assume these to take $\mathcal{O}(1)$ values. However, their values at the scale $M_c=1/R$ are enhanced by an additive contribution $\sim\ln(\Lambda/M_c)$, the coefficient of which is calculable in the low-energy effective field theory. Thus, the calculability of the Casimir energy is unspoiled as long as Λ and M_c are at least a few orders of magnitude apart.

The log-enhanced brane-operators discussed above affect the Kaluza–Klein mass spectrum, resulting in an extra contribution to the one-loop Casimir energy. This contribution is enhanced by $\ln(\Lambda R)$ because of the running of the brane-operators but suppressed by g^2/R because it is a brane-effect. Alternatively, this can be viewed as a two-loop effect since it arises in the interplay of the one-loop running and the one-loop Casimir energy calculation. This point of view is also consistent with the fact that this contribution is proportional to g^2. Due to the log-enhancement it dominates over the two-loop Casimir energy from the bulk (which is $\propto g^2/R^5$). In summary,

[4] For simplicity, we work with the effective potential in the Brans–Dicke frame, i.e., we do not absorb the prefactor R of the Einstein–Hilbert term into the metric. Since we will eventually only be interested in models with vanishing cosmological constant, this will not affect the position of the minimum.

[5] The Casimir energy

$$V(R_{\min})=\frac{c^{(1)}}{5R_{\min}^4}$$

is negative. In non-supersymmetric theories, one can simply introduce an appropriate brane tension (for models with branes or boundaries) in order to get a vanishing vacuum energy. In supersymmetric theories, this is less obvious, especially if one is not willing to compromise radion mediation as the dominant SUSY breaking mechanism. We will discuss this in detail in Section 7.

the 4d effective potential at leading two-loop order has the form

$$V(R) = \frac{1}{R^4}\left(c^{(1)} + c^{(\mathrm{br})}\ln(\Lambda R)\frac{g^2}{R}\right) = \frac{1}{R^4}\left(c^{(1)} + \tilde{c}^{(\mathrm{br})}\frac{\ln(\Lambda R)}{MR}\right), \tag{8}$$

where $c^{(\mathrm{br})}$ (or, equivalently, $\tilde{c}^{(\mathrm{br})}$) is the coefficient of the brane-induced contribution. This coefficient will be determined for supersymmetric gauge theories in Sections 3, 4 and 5.

As already discussed in the Introduction, we will assume that the cutoff scale Λ takes its highest possible value – the strong-coupling scale M of the 5d gauge theory (cf. Eq. (2)). Eq. (8) then determines the stabilization radius in terms of M and the calculable ratio $c^{(1)}/\tilde{c}^{(\mathrm{br})}$. This is the basis of our explicit determination of the unified gauge coupling.

Obviously, Λ is in principle an independent parameter of our 5d effective theory. For example, in an orbifold compactification of the heterotic string, Λ would depend on the values at which the dilaton and the 5 remaining compact dimensions are stabilized. As a further constraint, we would have to require that the 4d Planck mass is correctly reproduced. As discussed in some detail in [8], the present setting with a relatively large 5th dimension and a maximally extended validity range of the 5d gauge theory is one of the more appealing options for solving this complicated problem. This may be viewed as an extra motivation for our assumption $\Lambda \simeq M$.

Even if $\Lambda < M$, the main message of the present analysis remains unchanged: Eq. (8) will determine the compactification radius in terms of M, the field content, and Λ. The 'micro landscape' of orbifold GUTs will then allow us to tune the field content in such a way that a realistic 4d gauge coupling is obtained. In fact, Λ enters the Casimir energy only logarithmically and hence g_4^2 will also only have an (approximately) logarithmic dependence on Λ. Of course, our analysis breaks down if Λ is so small (i.e. the validity range of the 5d theory is so limited) that unknown $\mathcal{O}(1)$ terms are of the same size as $\ln(\Lambda/M_c)$.

3. Casimir energy for S^1/Z_2

Before determining the Casimir energy let us briefly review 5d $\mathcal{N}=1$ SUSY and its breaking by orbifold boundary conditions.

The 5d vector multiplet (VMP) consists of a real vector A_M, a real scalar Σ and a Dirac spinor λ, corresponding to two 4d Weyl spinors λ_L, λ_R. Under 4d $\mathcal{N}=1$ SUSY, it decomposes into a 4d vector multiplet $V = (A_\mu, \lambda_L)$ and a 4d chiral multiplet $\chi = (\Sigma + iA_5, \lambda_R)$. The 'gauginos' λ_L, λ_R can also be written as an $SU(2)_R$ doublet of symplectic Majorana spinors which makes the $SU(2)_R$ symmetry of the theory manifest [33]. The 5d hypermultiplet (HMP) consists of an $SU(2)_R$ doublet of scalars H^1, H^2 and a Dirac spinor ψ (which is equivalent to two Weyl spinors ψ_L and ψ_R). Under 4d $\mathcal{N}=1$ SUSY, it decomposes into a 4d chiral multiplet $H = (H^1, \psi_L)$ and another chiral multiplet $H^c = ((H^2)^*, \psi_R)$ in the conjugate representation of the gauge group.

The Z_2-parities of the fields can only be assigned consistently in a way that breaks 4d $\mathcal{N}=2$ SUSY to $\mathcal{N}=1$: Invariance of the action under Z_2 transformations demands V to be Z_2-even and χ to be Z_2-odd, while H and H^c must have opposite parities. Hence, only V and either H or H^c have Kaluza–Klein (KK) zero modes. The massive KK modes of the 5d VMP at each KK level form a 4d $\mathcal{N}=1$ massive vector multiplet (which has twice as many degrees of freedom (d.o.f.) as a massless 4d vector multiplet in Wess–Zumino gauge). On the other hand, the massive KK modes of H and H^c form pairs of massive 4d chiral multiplets.

The residual SUSY can be broken by a Scherk–Schwarz twist. It has been shown that this leads to the same spectrum as in radion mediated SUSY breaking [27,28,34]. Hence the latter

⋮	⋮	⋮	⋮	⋮	⋮	⋮	⋮	⋮
n=3	—	—↑—	—	—↑—	—↑—	—	—↑—	—
n=2	—	—↑—	—	—↑—	—↑—	—	—↑—	—
n=1	—	—↑—	—	—↑—	—↑—	—	—↑—	—
n=0	—	—↑—			—↑—	—		
KK-#	A_μ	λ_L	$\Sigma+iA_5$	λ_R	H^1	ψ_L	$(H^2)^*$	ψ_R

Fig. 1. KK masses of the components of the VMP and HMP. The arrows denote the states whose masses are shifted for $\omega \neq 0$.

scenario can be viewed as a dynamical realization of Scherk–Schwarz breaking. The bosons A_M, Σ in the VMP and the fermions ψ_L, ψ_R in the HMP are $SU(2)_R$ singlets and hence have 'untwisted' boundary conditions. On the other hand, the gauginos and hyperscalars are $SU(2)_R$ doublets which have a nontrivial twist-matrix $T = \exp(2\pi i \omega \sigma_2)$. Here, ω is the Scherk–Schwarz parameter (which can be identified with the radion F-term VEV) and σ_2 is the second Pauli matrix. As a consequence, the KK masses of the gauginos and hyperscalars receive a shift $n/R \to (n+\omega)/R$, which lifts the mass degeneracy of the 4d $\mathcal{N}=1$ SUSY multiplets (see Fig. 1). Even though the masses of bosons and fermions do not agree at any KK level, the UV-divergent part of the quantum corrections respects 4d $\mathcal{N}=1$ SUSY, which is *locally* unbroken. In other words, Scherk–Schwarz breaking is a global (= IR) effect which does not modify UV properties. In particular, the logarithmic divergences at the boundaries are supersymmetric, so that the resulting log-enhanced corrections to the KK masses are the same for all component fields within a 4d $\mathcal{N}=1$ supermultiplet.

To display the generic formula for the one-loop Casimir energy, we consider an $SU(2)_R$ doublet of complex scalars with opposite Z_2 parities on S^1/Z_2. Such a doublet forms the bosonic part of a hypermultiplet. The KK spectrum in the presence of a Scherk–Schwarz parameter ω (allowing also for $\omega = 0$) is $m_n^{(\omega)} \equiv (n+\omega)/R$. This gives a Casimir energy

$$\frac{1}{2} \times 4 \times \lim_{d \to 4} \frac{1}{2} \sum_{n=-\infty}^{\infty} \int \frac{d^d k_E}{(2\pi)^d} \ln\left(k_E^2 + \left(m_n^{(\omega)}\right)^2\right), \tag{9}$$

where the factor $1/2$ comes from the Z_2 projection, the factor 4 counts the d.o.f. of two complex scalars, and k_E is a Euclidean d-momentum. The finite, R-dependent part of the above expression is[6] [35]

$$4 f(\omega, R) \equiv -\frac{6\zeta_\omega(5)}{(2\pi)^6 R^4}, \tag{10}$$

where $\zeta_\omega(s) \equiv \sum_{k=1}^{\infty} k^{-s} \cos(2\pi k \omega)$ is a generalization of the Riemann zeta-function $\zeta_{\omega=0}$. In a theory with Scherk–Schwarz SUSY breaking, each d.o.f. with $\omega = 0$ has a superpartner with non-zero ω, so that the following quantity is useful:

$$c_\omega \equiv 2R^4\big(f(\omega, R) - f(0, R)\big) = \frac{3}{(2\pi)^6}\big(\zeta(5) - \zeta_\omega(5)\big). \tag{11}$$

Note that $c_\omega \geqslant 0$, since $\zeta(s) > \zeta_\omega(s)$ for $\omega \neq 0$.

[6] The one-loop vacuum diagrams can be split into an R-dependent finite part and a divergent part which is linear in R and represents a contribution to the 5d cosmological constant [14] (see also [36]). The latter cancels between bosons and fermions in supersymmetric models.

3.1. Casimir energy for a pure gauge theory

The one-loop bulk coefficient for a pure gauge theory with gauge group G and supergravity is obtained by adding the contributions from all physical d.o.f., taking into account a minus sign for fermions and the respective Scherk–Schwarz parameter ω of each field. One finds[7] [35,37]

$$c^{(1)} = c^{(1)}_{\text{vmp}} + c^{(1)}_{\text{grav}} \quad \text{where } c^{(1)}_{\text{vmp}} = -2c_\omega d_{\mathsf{G}},\ c^{(1)}_{\text{grav}} = -4c_\omega. \tag{12}$$

Let us now determine $c^{(\text{br})}$. This contribution comes from brane-localized operators. Such operators induce a shift δm_n of the KK masses. The relative mass shift $\Delta \equiv \delta m_n / m_n$ is independent of the KK level n. For one bosonic d.o.f. with tree level KK spectrum $m_n^{(\omega)}$, the extra contribution to $V(R)$ due to brane-localized operators is given by [23]

$$\begin{aligned} V^{(\text{br})}(R) &\equiv \frac{1}{2}\lim_{d\to 4}\frac{1}{2}\sum_{n=-\infty}^{\infty}\int\frac{d^d k_E}{(2\pi)^d}\left\{\ln\left(k_E^2 + (1+\Delta)^2\left(m_n^{(\omega)}\right)^2\right) - \ln\left(k_E^2 + \left(m_n^{(\omega)}\right)^2\right)\right\} \\ &= 4\Delta f(\omega, R) + \mathcal{O}\left(\Delta^2\right). \end{aligned} \tag{13}$$

To obtain the entire contribution to the Casimir energy, one has to sum over all d.o.f.

The brane-localized gauge-kinetic terms which are induced in case of a gauge theory are equivalent to a shift of the 4d effective gauge coupling g_4 by [11,30–32,38]

$$\delta\left(g_4^{-2}\right) = -\frac{1}{4\pi^2} C_{\mathsf{G}} \ln(MR). \tag{14}$$

This in turn corresponds to a relative shift of the KK masses of the VMP by

$$\Delta_{\text{vmp}} = -\frac{g^2}{2\pi R}\delta\left(g_4^{-2}\right) = \frac{1}{8\pi^3} C_{\mathsf{G}} \ln(MR)\frac{g^2}{R}. \tag{15}$$

Note that, in this equation, a factor $1/2$ arising from the fact that m^2 is corrected by $2\Delta_{\text{vmp}}$ cancels a factor 2 arising from the enhanced sensitivity of cosine-modes to brane terms as compared to the zero mode. Since Δ_{vmp} is the same for all d.o.f. of the VMP, using Eq. (13) and adding the contributions from all d.o.f. we find

$$V^{(\text{br})}_{\text{vmp}}(R) \equiv c^{(\text{br})}_{\text{vmp}} \ln(MR)\frac{g^2}{R^5} = 4\Delta_{\text{vmp}} c^{(1)}_{\text{vmp}} \frac{1}{R^4}. \tag{16}$$

Using this result and Eqs. (12) and (15) one can easily read off

$$c^{(\text{br})}_{\text{vmp}} = -\frac{c_\omega}{\pi^3} d_{\mathsf{G}} C_{\mathsf{G}}. \tag{17}$$

Note that this is always negative so that perturbatively controlled radius stabilization cannot be achieved in a pure super-Yang–Mills theory.

[7] Let us fix here our group theory conventions. The generators T^a_{R} for an irreducible representation R are normalized such that $\operatorname{tr}(T^a_{\mathsf{R}} T^b_{\mathsf{R}}) = C_{\mathsf{R}}\delta^{a,b}$, where C_{R} is the Dynkin index. The quadratic Casimir operator is denoted by $C_2(\mathsf{R})$. For the dimension of the representation R we use the notation d_{R}. The adjoint representation is denoted by $\mathsf{R} = \mathsf{G}$. We often use the identity $d_{\mathsf{R}} C_2(\mathsf{R}) = d_{\mathsf{G}} C_{\mathsf{R}}$.

3.2. Including hypermultiplets

Including the physical d.o.f. of HMPs, the one-loop bulk coefficient is [35,37]

$$c^{(1)} = c^{(1)}_{\mathrm{hmp}} + c^{(1)}_{\mathrm{vmp}} + c^{(1)}_{\mathrm{grav}} = 2c_\omega(d_{\mathsf{R}} - d_{\mathsf{G}} - 2). \tag{18}$$

We now determine $c^{(\mathrm{br})}$ in the presence of HMPs. As mentioned before, $c^{(\mathrm{br})}$ is due to brane-localized operators induced by quantum fluctuations above the compactification scale. More precisely, $c^{(\mathrm{br})}_{\mathrm{vmp}}$ respectively $c^{(\mathrm{br})}_{\mathrm{hmp}}$ denote the contribution from the one-loop selfenergy of the VMP respectively HMP. First of all, notice that, for both cases, the contribution from the HMP in the loop vanishes. To see why, recall that H and H^c have opposite Z_2 parities. Fields with opposite parities however lead to brane-localized operators of opposite signs.[8] Hence, the contributions from H and H^c cancel each other and only the VMP in the loop contributes to the selfenergy.

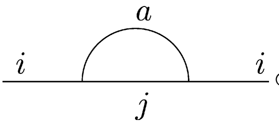
This means that $c^{(\mathrm{br})}_{\mathrm{vmp}}$, as given in Eq. (17), is unchanged. In the following we derive $c^{(\mathrm{br})}_{\mathrm{hmp}}$. To start with, let us consider a HMP in the adjoint representation. A 5d supersymmetric gauge theory with a VMP and an *adjoint* HMP has 5d $\mathcal{N} = 2$ SUSY, corresponding to $\mathcal{N} = 4$ in 4d.[9] Now, note that one of the four 4d SUSY parameters is invariant under modding out by reflections and translations: Two of the SUSY parameters are even under Z_2-reflections and one of these furthermore has no Scherk–Schwarz twist (cf. Fig. 1 where ψ_L is an invariant Weyl spinor). This means that after compactification on S^1/Z_2 the theory still has some unbroken SUSY so that the vacuum energy must vanish. Thus, the contribution of the adjoint HMP has to cancel that of the VMP:

$$c^{(\mathrm{br})}_{\mathrm{adj.hmp}} = \frac{c_\omega}{\pi^3} d_{\mathsf{G}} C_{\mathsf{G}}. \tag{19}$$

This corresponds to a KK mass shift $\Delta_{\mathrm{adj.hmp}} = \Delta_{\mathrm{vmp}}$.

Next we generalize this to a HMP in an arbitrary irreducible representation R. The only quantity that can change is the 'group theory factor' $d_{\mathsf{G}} C_{\mathsf{G}}$ in Eq. (19). Clearly, there is more than one expression for a general R which in the special case $\mathsf{R} = \mathsf{G}$ reduces to Eq. (19) (e.g. both $C_2(\mathsf{R})$ and C_{R} reduce to C_{G}). The correct generalization of Eq. (19) is found by recalling that the brane-localized operators arise from the one-loop selfenergy of the HMP with a VMP in the loop. Since the corresponding 'coupling matrices' are given by $(T^a_{\mathsf{R}})_{ij}$, the mass shift for the HMP component with gauge group index i (the result has to be independent of i, of course, due to the unbroken gauge symmetry) is

$$\Delta_{\mathrm{hmp}} \propto \sum_{a,j}$$



$$\propto \sum_a \left(T^a_{\mathsf{R}} T^a_{\mathsf{R}}\right)_{ii} \equiv C_2(\mathsf{R}). \tag{20}$$

Hence, the correct generalization of Eq. (19) is

$$c^{(\mathrm{br})}_{\mathrm{hmp}} = \frac{c_\omega}{\pi^3} d_{\mathsf{R}} C_2(\mathsf{R}) = \frac{c_\omega}{\pi^3} d_{\mathsf{G}} C_{\mathsf{R}}. \tag{21}$$

[8] The reason is the following [32]: For an S^1 compactification there are no 4d boundaries where logarithmic divergences can occur. As a consequence, logarithmic divergences due to the even KK modes have to be canceled by the divergences of the odd KK modes. Thus, on S^1/Z_2, fields with even and odd Z_2 parities give opposite log-divergences.

[9] This can for instance be verified by dimensional reduction of 10d supersymmetric gauge theory.

The total contribution of both a VMP and a HMP takes the simple form

$$c^{(\mathrm{br})} = \frac{c_\omega}{\pi^3} d_{\mathsf{G}} [C_{\mathsf{R}} - C_{\mathsf{G}}]. \tag{22}$$

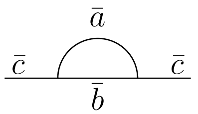
As an example, consider the case $G = SU(N)$ and h HMPs in the fundamental representation (so that $C_{\mathsf{R}} \to hC_{\mathsf{F}} = h/2$). One can easily check that for any number h it is impossible to have $c^{(1)} = 2c_\omega(hN - N^2 - 1) < 0$ and at the same time $c^{(\mathrm{br})} = c_\omega/\pi^3(N^2 - 1)[h/2 - N] > 0$. This situation may be improved if some of the gauge symmetry is broken. We discuss this in the following.

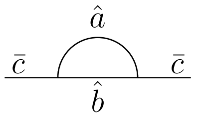
4. S^1/Z_2 with gauge symmetry breaking

Orbifold boundary conditions may break some of the bulk gauge symmetry at the boundaries [10]. Let us assume a breaking $G \to H = H_1 \otimes \cdots \otimes H_n$, where the H_i are the simple factors and $U(1)$ factors. The generators T^a of G are accordingly split into a set of generators $T^{\bar{a}}$ of H and a set of 'broken generators' $T^{\hat{a}}$. The HMP representation R of G splits into $\bigoplus_k \mathsf{R}_k$ where each R_k is a representation of $H = H_1 \otimes \cdots \otimes H_n$.

The one-loop bulk coefficient, Eq. (18), remains unchanged for S^1/Z_2 compactifications with broken gauge symmetry. The reason is simply that the KK mass spectrum is unchanged in comparison to the unbroken case (even though the wavefunctions of the higher KK modes of some components of the VMP and HMP have flipped Z_2-parity). As we will see in Section 5, this is different for $S^1/(Z_2 \times Z_2')$ compactifications, where also the one-loop bulk Casimir energy 'feels' the gauge symmetry breaking.

Since at the boundaries the gauge symmetry is broken, the boundary coefficient, Eq. (22), is modified. Let us first consider $c^{(\mathrm{br})}_{\mathrm{vmp}}$. As argued in Section 3, only the VMP in the loop gives a contribution. Z_2-invariance of the action implies that the structure constants f^{abc} have an even number of 'broken indices' [30]. The brane-localized operators leading to $c^{(\mathrm{br})}_{\mathrm{vmp}}$ are thus determined from diagrams with the following gauge group indices:

(1) 

(2) 

A crucial point is that the prefactor of the brane-localized terms induced by diagrams of type (1) is minus that of diagrams of type (2). The reason is the same which allowed us to argue that the contribution from the HMP in the loop vanishes: Fields with opposite parities give contributions of opposite signs. Thus, the mass shift of the unbroken VMP components with index $\bar{c}$ is (up to a prefactor which doesn't depend on group theory indices)

$$\Delta^{\bar{c}}_{\mathrm{vmp}} \propto + \sum_{\bar{a},\bar{b}} f^{\bar{a}\bar{b}\bar{c}} f^{\bar{a}\bar{b}\bar{c}} - \sum_{\hat{a},\hat{b}} f^{\hat{a}\hat{b}\bar{c}} f^{\hat{a}\hat{b}\bar{c}} = \sum_{a,b} \eta^a f^{ab\bar{c}} f^{ab\bar{c}}, \tag{23}$$

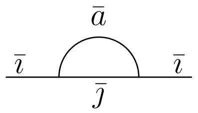
where we defined $\eta^{\bar{a}} = 1$, $\eta^{\hat{a}} = -1$. Since the tree-level KK masses of all d.o.f. are the same, one has

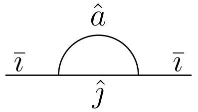
$$c_{\rm vmp}^{(\rm br)} \propto \sum_{\bar{c}} \Delta_{\rm vmp}^{\bar{c}} \propto \sum_{a,b,c} \eta^a f^{abc} f^{abc} = (2d_{\mathsf{H}} - d_{\mathsf{G}}) C_{\mathsf{G}}, \tag{24}$$

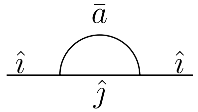
where we used $\sum_{\bar{a},\hat{b}} f^{\bar{a}\hat{b}\hat{c}} f^{\bar{a}\hat{b}\hat{c}} - \sum_{\hat{a},\bar{b}} f^{\hat{a}\bar{b}\hat{c}} f^{\hat{a}\bar{b}\hat{c}} = 0$. We thus have[10]

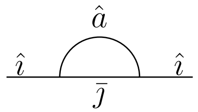
$$c_{\rm vmp}^{(\rm br)} = -\frac{c_\omega}{\pi^3} (2d_{\mathsf{H}} - d_{\mathsf{G}}) C_{\mathsf{G}}. \tag{25}$$

In a similar way we can infer $c_{\rm hmp}^{(\rm br)}$, which is due to the selfenergy of the HMP with a VMP in the loop. Let us split the indices $\{i\}$ of the HMP representation into two sets $\{\bar{\imath}\}$ and $\{\hat{\imath}\}$ by defining that, say, $H^{\bar{\imath}}$ is Z_2-even and $H^{\hat{\imath}}$ is Z_2-odd. Then, only elements $(T_{\mathsf{R}}^a)_{ij}$ with an even number of 'hatted indices' $\hat{a}$, $\hat{\imath}$ or $\hat{\jmath}$ are nonvanishing in order for the interaction terms to be Z_2-invariant. Thus, the group theory factor is determined by:

(1) 

(2) 

(3) 

(4) 

Note that, in contrast to VMP loops, the effect of HMP loops is non-zero even if the external index is Z_2-odd (cf. diagrams (3) and (4)).

For the same reason as above for $c_{\rm vmp}^{(\rm br)}$, (1) and (2), respectively (3) and (4), have prefactors of opposite signs. Thus we have

$$\Delta_{\rm hmp}^{\bar{\imath}} \propto + \sum_{\bar{a},\bar{\jmath}} \big(T_{\mathsf{R}}^{\bar{a}}\big)_{\bar{\imath}\bar{\jmath}} \big(T_{\mathsf{R}}^{\bar{a}}\big)_{\bar{\jmath}\bar{\imath}} - \sum_{\hat{a},\hat{\jmath}} \big(T_{\mathsf{R}}^{\hat{a}}\big)_{\bar{\imath}\hat{\jmath}} \big(T_{\mathsf{R}}^{\hat{a}}\big)_{\hat{\jmath}\bar{\imath}} = \sum_a \eta^a \big(T_{\mathsf{R}}^a T_{\mathsf{R}}^a\big)_{\bar{\imath}\bar{\imath}},$$

[10] In order to calculate $c_{\rm vmp}^{(\rm br)}$ one could also proceed as in Section 3 and use the shifts $\delta(g_{4,i}^{-2})$ of the 4d gauge couplings of the unbroken subgroups H_i, which can be extracted from Ref. [38]:

$$\delta\big(g_{4,i}^{-2}\big) = -\frac{1}{4\pi^2}\big(2C_2(\mathsf{H}_i) - C_2(\mathsf{G})\big)\ln(MR).$$

One arrives at the result

$$c_{\rm vmp}^{(\rm br)} = -\frac{c_\omega}{\pi^3}\big\{2\big(d_{\mathsf{H}_1} C_2(\mathsf{H}_1) + \cdots + d_{\mathsf{H}_n} C_2(\mathsf{H}_n)\big) - d_{\mathsf{H}} C_2(\mathsf{G})\big\}.$$

The above formula however depends on all factors H_i of H and hence conceals the fact that really the only information from the gauge symmetry breaking entering the result is the number d_{H}.

$$\Delta^{\hat{\imath}}_{\text{hmp}} \propto +\sum_{\bar{a},\hat{\jmath}} (T^{\bar{a}}_{\mathsf{R}})_{\hat{\imath}\hat{\jmath}} (T^{\bar{a}}_{\mathsf{R}})_{\hat{\jmath}\hat{\imath}} - \sum_{\hat{a},\bar{\jmath}} (T^{\hat{a}}_{\mathsf{R}})_{\hat{\imath}\bar{\jmath}} (T^{\hat{a}}_{\mathsf{R}})_{\bar{\jmath}\hat{\imath}} = \sum_{a} \eta^{a} (T^{a}_{\mathsf{R}} T^{a}_{\mathsf{R}})_{\hat{\imath}\hat{\imath}}. \tag{26}$$

Note that for $\eta^a = 1\ \forall a$, both expressions reduce to $C_2(\mathsf{R})$.

In order to calculate the Casimir energy, we furthermore need to know the (relative) prefactors which are missing in Eq. (26). To this end recall that H and H^c have opposite parities and H^c transforms under $\bar{\mathsf{R}}$ if H transforms under R. Thus, (1) corresponds to even modes in representation R (from $H^{\bar{\jmath}}$) and odd modes in rep. $\bar{\mathsf{R}}$ (from $H^{c\bar{\jmath}}$), while (3) corresponds to odd modes in rep. R (from $H^{\hat{\jmath}}$) and even modes in rep. $\bar{\mathsf{R}}$ (from $H^{c\hat{\jmath}}$). This shows that the interchange $\bar{\jmath} \leftrightarrow \hat{\jmath}$ corresponds to $\mathsf{R} \leftrightarrow \bar{\mathsf{R}}$. Since $(T^a_{\bar{\mathsf{R}}})_{ij} = -(T^a_{\mathsf{R}})_{ji}$, the proportionality factors between $\Delta^{\bar{\imath}}_{\text{hmp}}$, $\Delta^{\hat{\imath}}_{\text{hmp}}$ and the r.h. sides of Eq. (26) are the same. Thus we have

$$c^{(\text{br})}_{\text{hmp}} \propto +\sum_{\bar{\imath}} \Delta^{\bar{\imath}}_{\text{hmp}} + \sum_{\hat{\imath}} \Delta^{\hat{\imath}}_{\text{hmp}} \propto \sum_{a,i,j} \eta^{a} (T^{a}_{\mathsf{R}})_{ij} (T^{a}_{\mathsf{R}})_{ji} = (2d_{\mathsf{H}} - d_{\mathsf{G}}) C_{\mathsf{R}}. \tag{27}$$

The final result is then

$$c^{(\text{br})}_{\text{hmp}} = \frac{c_\omega}{\pi^3}(2d_{\mathsf{H}} - d_{\mathsf{G}}) C_{\mathsf{R}}. \tag{28}$$

Adding $c^{(\text{br})}_{\text{vmp}}$ and $c^{(\text{br})}_{\text{hmp}}$, we get the simple result

$$c^{(\text{br})} = \frac{c_\omega}{\pi^3}(2d_{\mathsf{H}} - d_{\mathsf{G}})[C_{\mathsf{R}} - C_{\mathsf{G}}]. \tag{29}$$

We stress that this does not depend on the details of the gauge symmetry breaking, but only on the dimension of H.

To illustrate this result we consider again $G = SU(N)$ and h fundamental HMPs. One has $C_{\mathsf{R}} - C_{\mathsf{G}} \to h/2 - N$ in this case. This is negative, unless $h \geqslant 2N$ which would however imply $c^{(1)} > 0$. Hence, $(2d_{\mathsf{H}} - d_{\mathsf{G}})$ needs to be negative in order to obtain $c^{(\text{br})} > 0$ and $c^{(1)} < 0$. The only possible breaking pattern of $SU(N)$ by Z_2 inner automorphisms is $SU(p+q) \to SU(p) \times SU(q) \times U(1)$ [39] (see also [40]), for which $2d_{\mathsf{H}} - d_{\mathsf{G}} = (p-q)^2 - 1$. This is negative only for $p = q$. A potentially phenomenologically interesting example for this is $SU(6) \to SU(3) \times SU(3) \times U(1)$ [41]. On the other hand, for the important case $SU(5) \to SU(3) \times SU(2) \times U(1)$, the term $(2d_{\mathsf{H}} - d_{\mathsf{G}})$ is zero, i.e. the contribution of the bulk fields to the leading order two-loop term vanishes.

5. $S^1/(Z_2 \times Z_2')$ with broken gauge symmetry at one brane

The space $S^1/(Z_2 \times Z_2')$ is obtained by modding out by a second Z_2' parity. In this case, fields can have different boundary conditions at the two inequivalent fixed points. If some of the gauge symmetry is broken at both of the boundaries, there are additional massless fields at tree level besides the radion, namely the zero modes of some of the higher-dimensional components of the gauge bosons.[11] They acquire masses from radiative corrections [42]. In this section we restrict our attention to situations where the gauge symmetry remains unbroken at one of the branes. In that case the radion is the only modulus.

[11] This is also the case for S^1/Z_2 compactification with broken gauge symmetry.

5.1. Pure gauge theory

As in the previous sections we start with a pure gauge theory. The (Z_2, Z_2')-parities and the KK levels of the 4d superfields (V, χ) which form the VMP are shown in the following table (cf. [11,30,31]):

d.o.f.	(Z_2, Z_2')-parity	KK spectrum
$V^{\bar{a}}$	$(+,+)$	$2n/R$
$V^{\hat{a}}$	$(+,-)$	$(2n+1)/R$
$\chi^{\bar{a}}$	$(-,-)$	$(2n+2)/R$
$\chi^{\hat{a}}$	$(-,+)$	$(2n+1)/R$

Note that the KK masses of broken and unbroken components of a multiplet are displaced. This is an important difference to the S^1/Z_2 case, which has the consequence that also the one-loop bulk Casimir energy feels the breaking of the gauge symmetry as we will see.

The Casimir energy for one bosonic d.o.f. on $S^1/(Z_2 \times Z_2')$ with KK spectrum $2(n+\omega)/R$ is given by $f(\omega, R/2)$ while for one d.o.f. with KK spectrum $[2(n+\omega)+1]/R$ it is $f(\omega+1/2, R/2)$. Using the duplication formula $Li_s(z) + Li_s(-z) = 2^{1-s} Li_s(z^2)$ for polylogarithms $Li_s(z) \equiv \sum_{k=1}^{\infty} k^{-s} z^k$, one finds $\zeta_{\omega+\frac{1}{2}}(5) = -\zeta_\omega(5) + 1/16\zeta_{2\omega}(5)$. This, together with the inverse quartic scaling of $f(\omega, R)$ with R, results in

$$\begin{aligned} &f(\omega, R/2) = +16 f(\omega, R), \\ &f(\omega+1/2, R/2) = -16 f(\omega, R) + f(2\omega, R). \end{aligned} \tag{30}$$

Fields with odd KK spectrum give an almost opposite contribution to the Casimir energy as fields with even KK spectrum.

By adding the contributions from all d.o.f. – taking into account the boundary conditions and spin of each d.o.f. – one finds with the help of Eq. (30) that

$$c_{\text{vmp}}^{(1)} = -32(2d_{\mathsf{H}} - d_{\mathsf{G}})c_\omega - 2(d_{\mathsf{G}} - d_{\mathsf{H}})c_{2\omega}. \tag{31}$$

As a simple check, for the unbroken case $\mathsf{H} = \mathsf{G}$ this becomes $c_{\text{vmp}}^{(1)} = -32 d_{\mathsf{G}} c_\omega$, which is 16 times the result for S^1/Z_2. The relative factor of $16(=2^4)$ arises, because the length of the physical space for $S^1/(Z_2 \times Z_2')$ is one half of that for S^1/Z_2.

We now determine the brane-coefficient. The UV-divergent contribution to the brane-localized operators is induced by fluctuations of the bulk fields 'close to' the brane. It can therefore not depend on the boundary conditions at the other brane. This implies that for the unbroken brane of $S^1/(Z_2 \times Z_2')$ we can use the mass shift for S^1/Z_2 with unbroken gauge symmetry (cf. Eq. (15)), and for the broken brane we can use the mass shift for S^1/Z_2 with broken gauge symmetry (cf. Eq. (23)). More precisely, we have to take one half of Eq. (15) respectively Eq. (23), since we need the contribution of only one brane. The argument of this paragraph is also valid for the HMP contribution discussed below.

The gauge coupling correction, or equivalently the mass shift of the KK modes (Eq. (15)), which determines the coefficient $c_{\text{vmp}}^{(\text{br},G)}$ for the unbroken brane, is G-universal. Thus, Eq. (16) applies, and together with Eq. (31) we immediately get

$$c_{\text{vmp}}^{(\text{br},G)} = -\frac{8}{\pi^3} C_{\mathsf{G}} \left[(2d_{\mathsf{H}} - d_{\mathsf{G}})c_\omega + (d_{\mathsf{G}} - d_{\mathsf{H}}) \frac{c_{2\omega}}{16} \right]. \tag{32}$$

In order to determine the coefficient $c_{\text{vmp}}^{(\text{br},H)}$ for the brane where the gauge symmetry is reduced to H, we use Eq. (23). Moreover, since no brane-localized terms are induced for the broken components of the VMP (i.e. those with 'shifted' KK spectrum $(2n+1)/R$) at that brane, the summation Eq. (24) still applies and we get

$$c_{\text{vmp}}^{(\text{br},H)} = -\frac{8c_\omega}{\pi^3} C_{\mathsf{G}}(2d_{\mathsf{H}} - d_{\mathsf{G}}). \tag{33}$$

The factor 8 in comparison to Eq. (25) arises, since we have to multiply by 2^4 (for the reduced length) and divide by two (for *one* brane).

5.2. Including hypermultiplets

The (Z_2, Z_2')-parities of the 4d superfields H and H^c which form the HMP follow from the (Z_2, Z_2')-parities of the VMP. H and H^c necessarily have opposite Z_2-parities and opposite Z_2'-parities, leading to a chiral spectrum for the zero modes. The Z_2'-parities depend on the gauge group index. We still have the freedom to choose an overall sign of the Z_2' action on the HMP. Similar to the S^1/Z_2-case, we define, by the following table, two sets of indices $\{\bar\imath\}$ and $\{\hat\imath\}$:

d.o.f.	(Z_2, Z_2')-parity	KK spectrum
$H^{\bar\imath}$	$(+,+)$	$2n/R$
$H^{\hat\imath}$	$(+,-)$	$(2n+1)/R$
$H^{c\bar\imath}$	$(-,-)$	$(2n+2)/R$
$H^{c\hat\imath}$	$(-,+)$	$(2n+1)/R$

The number of $\bar\imath$-indices is denoted by d_1 and the number of $\hat\imath$-indices is denoted by d_2, so that $d_1 + d_2 = d_{\mathsf{R}}$. Using Eq. (30), the bulk coefficient for the HMP is found to be

$$c_{\text{hmp}}^{(1)} = 32(d_1 - d_2)c_\omega + 2d_2 c_{2\omega}. \tag{34}$$

The brane-coefficient from the unbroken brane, denoted by $c_{\text{hmp}}^{(\text{br},G)}$, follows from the results of Section 3: From Eqs. (18) and (21), together with Eq. (16), one reads off that (for one brane)

$$\Delta_{\text{hmp}} = \frac{1}{16\pi^3} C_2(\mathsf{R}) \ln(MR)\frac{g^2}{R}. \tag{35}$$

This and Eq. (16) lead to

$$c_{\text{hmp}}^{(\text{br},G)} = \frac{8}{\pi^3} C_2(\mathsf{R})\left[(d_1 - d_2)c_\omega + d_2\frac{c_{2\omega}}{16}\right]. \tag{36}$$

Regarding the contribution $c_{\text{hmp}}^{(\text{br},H)}$, Eq. (26) together with the subsequent paragraph implies (for $i = \bar\imath$ and $i = \hat\imath$)

$$\Delta_{\text{hmp}}^{i} = \frac{1}{16\pi^3} \sum_a \eta^a \left(T_{\mathsf{R}}^a T_{\mathsf{R}}^a\right)_{ii} \ln(MR)\frac{g^2}{R}. \tag{37}$$

Using this and Eq. (13), one arrives at

$$c_{\text{hmp}}^{(\text{br},H)} = \frac{8}{\pi^3}\left[\left(\sum_{a,\bar\imath} \eta^a \left(T_{\mathsf{R}}^a T_{\mathsf{R}}^a\right)_{\bar\imath\bar\imath} - \sum_{a,\hat\imath} \eta^a \left(T_{\mathsf{R}}^a T_{\mathsf{R}}^a\right)_{\hat\imath\hat\imath}\right)c_\omega + \left(\sum_{a,\hat\imath} \eta^a \left(T_{\mathsf{R}}^a T_{\mathsf{R}}^a\right)_{\hat\imath\hat\imath}\right)\frac{c_{2\omega}}{16}\right]. \tag{38}$$

As a simple check, one can verify that for an adjoint HMP, the expression $\sum_{a,\hat{\imath}} \eta^a (T^a_{\mathsf{R}} T^a_{\mathsf{R}})_{\hat{\imath}\hat{\imath}}$ vanishes, so that in this case one indeed has $c^{(\mathrm{br},H)}_{\mathrm{hmp}} = -c^{(\mathrm{br},H)}_{\mathrm{vmp}}$.

The above results (34), (36) and (38) depend on d_1 and d_2. This dependence disappears if one has a second HMP with opposite Z_2'-parities, but the same quantum numbers. The flip of the Z_2'-parities corresponds to an interchange of d_1 and d_2. Then Eqs. (34), (36) and (38), for the combined effect of such a pair of HMPs, simplify to

$$\begin{aligned}
c^{(1)}_{\mathrm{hmp+hmp'}} &= 2d_{\mathsf{R}} c_{2\omega}, \\
c^{(\mathrm{br},G)}_{\mathrm{hmp+hmp'}} &= \frac{1}{2\pi^3} d_{\mathsf{G}} C_{\mathsf{R}} c_{2\omega}, \\
c^{(\mathrm{br},H)}_{\mathrm{hmp+hmp'}} &= \frac{1}{2\pi^3} (2d_{\mathsf{H}} - d_{\mathsf{G}}) C_{\mathsf{R}} c_{2\omega}.
\end{aligned} \tag{39}$$

6. Application to 5d SUSY-GUT models

For the application of our results to realistic models, we need to consider also the effect of charged chiral multiplets located at a boundary (see e.g. [33] for an explicit Lagrangian). This can for instance be an MSSM matter- or Higgs-sector. The inclusion of their effects in the Casimir energy is straightforward: The contribution of brane fields to the running of the gauge coupling is the usual one of 4d gauge theories. For fields at a brane (of an S^1/Z_2) where the gauge symmetry is $H = H_1 \otimes \cdots \otimes H_n$ this yields[12]

$$c^{(\mathrm{br})}_{\mathrm{loc}} = \frac{c_\omega}{2\pi^3} \sum_{i=1}^{n} d_{\mathsf{H}_i} b_{H_i}. \tag{40}$$

Here, b_{H_i} is the β-function coefficient for the unbroken subgroup H_i. For the case of a brane with unbroken gauge symmetry this becomes $c^{(\mathrm{br})}_{\mathrm{loc}} = (c_\omega/2\pi^3) d_{\mathsf{G}} b_G$. For $S^1/(Z_2 \times Z_2')$, the only difference is an extra factor of 16 due to the reduced length of the interval. Observe that $c^{(\mathrm{br})}_{\mathrm{loc}}$ is always positive since the β-function coefficient of chiral multiplets is positive. Hence, in situations where the contribution of the bulk field content alone to $c^{(\mathrm{br})}$ is not positive and large as it needs to be, brane-localized chiral multiplets can help to assure a perturbatively controlled radion effective potential.

Let us now apply our results to supersymmetric $SU(5)$-GUTs on $S^1/(Z_2 \times Z_2')$ such as those which were proposed in Ref. [11] (see also [10]). These models have an unbroken $SU(5)$ brane as well as a brane where the gauge symmetry is broken to the Standard Model (SM) gauge group $SU(3) \times SU(2) \times U(1)$. The gauge sector resides in the bulk and the Higgs sector is located on the SM brane (this avoids the doublet-triplet splitting problem). In Ref. [11], the matter sector is assumed to be located either completely on the SM brane or in the bulk. As explained in detail in Ref. [11], in the latter case each bulk matter family consists of *two* copies of a $(\mathbf{10} + \bar{\mathbf{5}})$ with opposite Z_2'-parities, such that there is a full MSSM matter family at the zero mode level.

We generalize the models of Ref. [11] by allowing for an arbitrary distribution of the MSSM matter to the bulk and the SM brane, which is our 'micro-landscape'. For any of the three families, we allow the freedom to have, instead of a pair with opposite Z_2'-parities, just one copy

[12] In our conventions, the β-function coefficient for fields charged under a gauge group G is $b_G = \frac{1}{6}[(-22)C_{\mathsf{G}} + 4(\#\text{ of Weyl-fermions in rep. }\mathsf{R})C_{\mathsf{R}} + 2(\#\text{ of complex scalars in rep. }\mathsf{R}')C_{\mathsf{R}'}]$.

of a $\bar{\mathbf{5}} = (\bar{\mathbf{3}}, \mathbf{1})_{\mathbf{1/3}} \oplus (\mathbf{1}, \mathbf{2})_{-\mathbf{1/2}}$ in the bulk. Depending on the Z_2'-parity of this $\bar{\mathbf{5}}$, there is either a $D_R = (\bar{\mathbf{3}}, \mathbf{1})_{\mathbf{1/3}}$ or an $L_L = (\mathbf{1}, \mathbf{2})_{-\mathbf{1/2}}$ at the zero mode level. Analogously, for one HMP transforming as a $\mathbf{10} = (\mathbf{3}, \mathbf{2})_{\mathbf{1/6}} \oplus (\bar{\mathbf{3}}, \mathbf{1})_{-\mathbf{2/3}} \oplus (\mathbf{1}, \mathbf{1})_{\mathbf{1}}$, one has either a $Q_L = (\mathbf{3}, \mathbf{2})_{\mathbf{1/6}}$ or a $U_R \oplus E_R = (\bar{\mathbf{3}}, \mathbf{1})_{-\mathbf{2/3}} \oplus (\mathbf{1}, \mathbf{1})_{\mathbf{1}}$ at the zero mode level. Let us in the following consider the most general situation where the zero modes of HMPs lead to r generations of $U_R \oplus E_R$, s generations of L_L, t generations of Q_L and u generations of D_R in the bulk (where $r, s, t, u \in \{0, 1, 2, 3\}$). Consequently, the remaining $(3 - r)$ $U_R \oplus E_R$ generations, $(3 - s)$ L_L generations, $(3 - t)$ Q_L generations and $(3 - u)$ D_R generations must be located at the SM brane.

We now determine the Casimir energy for this situation. Applying Eqs. (31) and (34) and adding $c_{\text{grav}}^{(1)} = -64c_\omega$ from the supergravity multiplet, one finds

$$\frac{c^{(1)}}{c_\omega} = -160 - 16r - 8s + 96t + 48u, \tag{41}$$

where we used $c_{2\omega} = 4c_\omega + \mathcal{O}(\omega^4)$. Here and in the following we neglect $\mathcal{O}(\omega^4)$ effects. The brane contribution to the Casimir energy which is induced by the VMPs and HMPs in the bulk is given by Eqs. (32), (33), (36) and (38).[13] Adding all terms, one finds after some algebra that

$$\frac{c_{\text{hmp+vmp}}^{(\text{br})}}{c_\omega} = -\frac{12}{5\pi^3}(50 + 27r + 9s - 57t - 19u). \tag{42}$$

On the other hand, using Eq. (40), the brane effect induced by the matter and Higgs fields on the brane is found to be

$$\frac{c_{\text{loc}}^{(\text{br})}}{c_\omega} = \frac{24}{5\pi^3}(126 - 9r - 3s - 21t - 7u), \tag{43}$$

so that the total brane coefficient is

$$\frac{c^{(\text{br})}}{c_\omega} = \frac{1}{\pi^3}(2424/5 - 108r - 36s + 36t + 12u). \tag{44}$$

One can now determine the position of the minimum of $V(R)$ in units of g^2 or equivalently in units of $1/M \simeq g^2 N/(24\pi^3)$, for all choices of r, s, t, u. Note that the condition $c^{(1)} < 0$ for the existence of a minimum is not satisfied for many choices of r, s, t, u, while $c^{(\text{br})} > 0$ is always satisfied. If a minimum exists, one easily finds that it is given by

$$MR = \gamma W_{-1}\left(\sqrt[5]{e}/\gamma\right) \quad \text{where } \gamma \equiv \frac{5}{4}\frac{24\pi^3}{N}\frac{c^{(\text{br})}}{c^{(1)}}. \tag{45}$$

[13] To evaluate Eq. (36) one needs $C_2(\bar{\mathbf{5}}) = 12/5$ and $C_2(\mathbf{10}) = 18/5$. To evaluate Eq. (38) one needs

$$\sum_{a,\bar{\imath}} \eta^a \left(T^a_{\bar{\mathbf{5}}} T^a_{\bar{\mathbf{5}}}\right)_{\bar{\imath}\bar{\imath}} = -6/5, \qquad \sum_{a,\hat{\imath}} \eta^a \left(T^a_{\bar{\mathbf{5}}} T^a_{\bar{\mathbf{5}}}\right)_{\hat{\imath}\hat{\imath}} = 6/5,$$

where $\bar{\imath}$ is an L_L-index and $\hat{\imath}$ a D_R-index, as well as

$$\sum_{a,\bar{\imath}} \eta^a \left(T^a_{\mathbf{10}} T^a_{\mathbf{10}}\right)_{\bar{\imath}\bar{\imath}} = -18/5, \qquad \sum_{a,\hat{\imath}} \eta^a \left(T^a_{\mathbf{10}} T^a_{\mathbf{10}}\right)_{\hat{\imath}\hat{\imath}} = 18/5,$$

where $\bar{\imath}$ is a $U_R \oplus E_R$-index and $\hat{\imath}$ a Q_L-index.

Here, $W_{-1}(x)$ is the Lambert W-function, which is defined as the inverse function of xe^x. More precisely, since the Lambert W-function is double-valued on $(-1/e, 0)$, $W_{-1}(x)$ denotes the branch which satisfies $W_{-1}(x) \leqslant -1$ for $-1/e \leqslant x < 0$ (see e.g. [43]). By scanning all $4^4 = 256$ possible choices of r, s, t, u, we find that $V(R)$ has a minimum at large radius, say $MR > 10$, for about a third of them.

Since the minimum R is given in units of $1/M$ (or equivalently g^2, as explained in the Introduction), by stabilizing the radius we determine the value of the 4d gauge coupling $g_4^2 = g^2/(2\pi R)$ at the scale M_c. The phenomenological value is $\alpha(M_c) = g_4^2/4\pi \simeq 1/25$. For 12 choices of r, s, t, u, we find values for $\alpha^{-1}(M_c)$ in the interval 20–30 (we give rounded values):

r	1	1	2	2	0	2	1	1	3	0	3	2
s	1	3	2	0	3	2	0	2	1	2	1	1
t	0	0	0	0	0	1	0	0	0	0	1	0
u	0	1	2	1	0	0	0	1	3	0	1	2
α^{-1}	20	20	22	22	23	24	25	26	26	28	29	30

For some orbifold GUT models (albeit not the most simple ones) we predict a realistic size for the unified gauge coupling!

As a comparison, for the most simple model where all matter is located at the SM brane (i.e. $r = s = t = u = 0$) we find $\alpha \simeq 1/40$. This is too small. One should keep in mind however that the correction due to unknown nonvanishing but not unnaturally large coefficients of the brane-localized operators at the scale M is expected to be roughly of the order $1/\ln(MR) \sim 25\%$.

7. Uplifting to a small cosmological constant

The Casimir energy we find gives a negative contribution of the order $\omega^2/R^4 \sim m_{1/2}^2 M_c^2$ to the vacuum energy. In order for the theory to be potentially realistic, there need to be other effects canceling this negative contribution, such that a tiny positive Λ_4 is obtained. This will certainly involve fine-tuning, otherwise we would have solved the cosmological constant problem.[14] In this section, we briefly discuss how such an 'uplifting' could be realized. We also check that our proposals are consistent with the assumptions of our Casimir energy calculation, namely a (sufficiently) flat 5d background and Scherk–Schwarz SUSY breaking.

Note that there are no loop-contributions to Λ_4 coming from fields other than the radion. This is easily seen in the language of 4d supergravity: Our Casimir energy calculation is equivalent to the calculation of a loop correction to the no-scale Kähler potential of the radion. In the presence of a constant brane-localized superpotential W_0, which is related to the Scherk–Schwarz

[14] Given our string-theoretic motivation, which relies mainly on the recent progress in heterotic orbifold model building, it is tempting to ascribe the required fine tuning to the multitude of vacua in the string theory landscape. The problem with this argument is our insufficient understanding of the heterotic landscape, which at present does not allow us to find a sufficiently large and dense discretuum within the relevant orbifold constructions (unlike the type IIB case, where the situation is more promising from the perspective of the cosmological constant). For the purpose of this paper, we take the optimistic attitude that either such a heterotic landscape will be found or the cosmological constant problem will be solved in some other way.

parameter ω by

$$\omega \sim \frac{|W_0|}{M_{P,5}^3}, \tag{46}$$

this correction turns into a potential energy. Loop effects will generically also correct the Kähler potential of matter and Higgs fields, which is canonical at tree level. Since these fields do not develop a VEV, their Kähler corrections do not induce a contribution to the vacuum energy. Furthermore, there are no perturbative corrections to the superpotential. Thus, the negative vacuum energy which we find at the minimum of the radion effective potential has to be taken seriously and some compensating effect is required.

7.1. Uplifting by small warping and a brane-superpotential

Let us allow for a 5d cosmological constant, which of course has to be small enough not to affect our flat-space Casimir energy calculation. In other words, we assume that we are dealing with a supersymmetric Randall–Sundrum model [44], but with very weak warping. The drawback of this proposal is that we do not know how such a warping could arise from the heterotic orbifold perspective. One may hope that it can be realized, e.g., by fluxes in the five compact dimensions, which would make it discretely tunable. In any case, what follows should be consistent from the point of view of 5d supergravity coupled to gauge fields and charged matter.

In the presence of a constant superpotential W_0 at the IR brane, the corresponding 4d theory is defined by [45]

$$\Omega = \frac{3M_{P,5}^3}{k}\left(e^{-k(T+\bar{T})} - 1\right) \quad \text{and} \quad W = W_0 e^{-3kT}. \tag{47}$$

The AdS curvature scale k is related to the 5d cosmological constant by $\Lambda_5 = -6k^2 M_{P,5}^3$. Working on S^1/Z_2, the $T = \pi R + \cdots$ is the radion superfield, while Ω and W are the 'superspace kinetic function' and the superpotential of 4d supergravity. A Kähler–Weyl rescaling brings them to the equivalent form

$$\Omega = \frac{3M_{P,5}^3}{k}\left(1 - e^{k(T+\bar{T})}\right) \equiv -3M_{P,5}^3(T+\bar{T}) + \Delta\Omega \quad \text{and} \quad W = W_0. \tag{48}$$

For weak warping, $k(T+\bar{T}) \ll 1$, we have

$$\Delta\Omega \simeq -\frac{3}{2}M_{P,5}^3 k(T+\bar{T})^2, \tag{49}$$

which can then be treated as a small correction to the basic no-scale structure of the model. This puts us into the setting of 'Almost no-scale supergravity' of Luty and Okada [17], where the corresponding correction to the Brans–Dicke-frame scalar potential,

$$\delta V = -\frac{|W_0|^2}{M_{P,5}^6}(\Delta\Omega)_{T\bar{T}} = 3k\frac{|W_0|^2}{M_{P,5}^3}, \tag{50}$$

has been given.[15] One immediately sees that, in order for δV to cancel the negative Casimir energy, one needs a warping of the order

$$k(T+\bar{T}) \sim \left(\frac{M_c}{M_{P,5}}\right)^3. \tag{51}$$

Thus, the warping required for the uplifting is indeed small whenever the stabilization radius is large in units of the 5d Planck scale (this is anyway necessary for gravity to be perturbative at the compactification scale). We conclude that our flat space calculation remains justified in spite of the fact that we are really dealing with a Randall–Sundrum type model.

We finally note that our complete stabilization and uplifting proposal can be formulated within the framework of [17]: Our two-loop Casimir energy can be reinterpreted as a Kähler correction with the structure

$$\Delta\Omega_{\text{Casimir}} \sim \frac{1}{(T+\bar{T})^2} + \frac{g^2}{(T+\bar{T})^3}\ln\bigl(M(T+\bar{T})\bigr). \tag{52}$$

Adding this to the warping-induced correction of Eq. (49), we find that Eq. (50) generates a scalar potential the minimum of which can be tuned to zero by adjusting the ratios of k, $M_{P,5}$ and g.

7.2. Uplifting in a detuned Randall–Sundrum model

The uplifting proposal of the last subsection can be reformulated in terms of the 'supersymmetric detuned Randall–Sundrum model' of Bagger and Belyaev [46]. According to [46], the UV and IR brane tensions Λ_0 and Λ_π, which are normally given by

$$\Lambda_0 = -\Lambda_\pi = \sqrt{-6\Lambda_5 M_{P,5}^3} \equiv \Lambda, \tag{53}$$

can take arbitrary values in a consistent 5d supergravity model, as long as they obey the constraint $|\Lambda_{0,\pi}| \leqslant \Lambda$. Thus, it is natural to attempt to uplift our previous stabilized flat 5d model by allowing for a small warping together with a small detuning of the IR-brane tension,

$$0 < \Lambda_\pi + \Lambda \ll \Lambda. \tag{54}$$

We keep $\Lambda_0 = \Lambda$ for simplicity.

The naive expectation is that, as long as warping is small, this will give a constant and positive contribution to the radion effective potential in the Brans–Dicke frame. Note that this is not inconsistent with general theorems concerning the possible vacuum states of supergravity theories: The detuned Randall–Sundrum model has an AdS_4 ground state at a certain radius. Since we stabilize the radius by the Casimir energy at a different value, we are actually forcing the theory into a metastable state with a tiny positive Λ_4.

To confirm the above expectation, we utilize the 4d supergravity description of the detuned Randall–Sundrum model derived in [47]. The Kähler potential K and the superpotential W are explicitly given in Eqs. (6.1) of [47]. We first rewrite K in terms of $\Omega = -3\exp(-K/3)$ and perform a (constant) Kähler–Weyl transformation bringing Ω to the form given in Eq. (47). We

[15] Beware of a typo in Eq. (6) of the arXiv-version of [17].

then work out W in the limit $\Lambda_0 \to \Lambda$, making also use of the relation $\Lambda_\pi + \Lambda \ll \Lambda$. The functional form agrees with our Eq. (47) and we determine

$$|W_0| = \sqrt{2} M_{P,5}^3 \sqrt{1 + \Lambda_\pi / \Lambda}. \tag{55}$$

Calculating δV according to Eq. (50), we find

$$\delta V = \Lambda_\pi + \Lambda, \tag{56}$$

as expected (see also [48]). We conclude that the uplifting proposals of Section 7.1 and of the present section are equivalent. The underlying technical result is the superfield formulation of the detuned Randall–Sundrum model of [47]: It implies that including a constant IR-brane-localized superpotential in a supersymmetric Randall–Sundrum model is equivalent to a weak detuning of the IR brane tension. In both cases, a non-zero scalar potential is induced. This potential is positive and approximately constant at small values of the radion (i.e. for a warp factor close to one).

We note that the 4d superfield description of the detuned Randall–Sundrum model has also been considered, e.g., in [48] (independently of [47]) as well as in [49] and [21]. In particular, one-loop corrections to the radion potential have been analyzed in [48,49]. Our 'uplifting' proposal of Sections 7.1 and 7.2 differs in that we use a two-loop effective potential to stabilize the radion at a very small (from the Randall–Sundrum model perspective) value. The warping is then irrelevant for the loop calculation and its only effect is to provide, in its interplay with a small detuning, an approximately constant uplifting contribution. For related applications of the detuning in supersymmetric Randall–Sundrum models see, e.g., [19,50].

7.3. *F-term uplifting*

Alternatively, one may insist that Λ_5 is exactly zero. In that case, Scherk–Schwarz breaking predicts a negative vacuum energy and there needs to be another source of SUSY breaking, in the spirit of 'F-term uplifting' (see e.g. [51]). Following again [17], we assume that via some unspecified dynamics there arises an F-term VEV of a brane-localized singlet S of the right order of magnitude to obtain a small cosmological constant: $F_S \sim \omega/R^2$. This can potentially affect the SUSY-breaking mass splitting of bulk fields and hence our Casimir energy calculation.

In the spirit of gaugino mediation, S may couple to the gauge-kinetic term via a brane-localized higher-dimension operator, suppressed by the fundamental scale M. The induced gaugino masses are of the order [52]

$$m_{1/2} \sim \frac{F_S}{M^2 R} \sim \frac{\omega}{M^2 R^3}. \tag{57}$$

Similarly, S may couple to the kinetic term of bulk hypermultiplets via a higher-dimension brane-localized operator. This induces scalar masses of the same order of magnitude as the gaugino masses given in Eq. (57), $m_0 \sim m_{1/2}$.

By contrast, the 'radion-mediated contribution' to the gaugino masses and to the scalar masses of bulk multiplets is of the order

$$m_0 \sim m_{1/2} \sim \frac{\omega}{R}, \tag{58}$$

so that in comparison the effect of F_S is suppressed by $1/(MR)^2$. We conclude that our Casimir energy calculation and the corresponding stabilization mechanism remain under quantitative control in the presence of brane-localized F-term uplifting. One may nevertheless feel that the uplifting mechanism of Sections 7.1 and 7.2 is more elegant since it does not require an extra SUSY breaking sector.

8. Conclusions

We have analyzed Casimir stabilization of supersymmetric gauge theories on 5d orbifolds with Scherk–Schwarz SUSY breaking and gauge symmetry breaking by boundary conditions. Depending on field content and symmetries of the 5d theory, a minimum in the Casimir energy (as a function of the radius R) can arise from the interplay of the one-loop and two-loop contribution. The dominant two-loop effect comes from logarithmically divergent operators localized at the boundaries. We rely only on the log-enhanced part of the coefficients of these operators. Our results are therefore independent of the UV completion as long as the compactification scale $1/R$ is much smaller than the cutoff scale Λ. Then, the Casimir energy – including the one-loop term – is given by (see Eq. (8))

$$V(R) = \frac{1}{R^4}\left(c^{(1)} + c^{(\mathrm{br})}\ln(\Lambda R)\frac{g^2}{R}\right). \tag{59}$$

We provide general formulae for the one-loop coefficient $c^{(1)}$ and for the coefficient $c^{(\mathrm{br})}$ of the brane-induced two-loop effect. The coefficient $c^{(1)}$ is well known for the case of unbroken gauge symmetry (Eq. (18)). It is unchanged for S^1/Z_2 with broken gauge symmetry. For $S^1/(Z_2 \times Z_2')$ with gauge symmetry broken at one of the branes, it is given by Eq. (31) for vector- and by Eq. (34) for hypermultiplets.

For S^1/Z_2, the coefficient $c^{(\mathrm{br})}$ is given by Eq. (22) for unbroken gauge symmetry and by Eq. (29) for gauge symmetry breaking by boundary conditions. In the most relevant case of $S^1/(Z_2 \times Z_2')$ with gauge symmetry breaking at one of the boundaries, $c^{(\mathrm{br})}$ is given by Eqs. (32), (33) for vector multiplets and by Eqs. (36), (38) for hypermultiplets. We were able to obtain these coefficients without explicit new loop calculations, relying only on the Z_2 parity transformation properties of the fields, supersymmetry and the group theoretic structure of the model.

We have applied the above formulae to $SU(5)$ orbifold GUT models on $S^1/(Z_2 \times Z_2')$, with gauge-symmetry breaking to the Standard Model at one of the boundaries. For simplicity, we have assumed that the cutoff Λ takes its highest possible value – the strong coupling scale $M \simeq 24\pi^3/(5g^2)$ of the 5d gauge theory. Furthermore, we have focused on scenarios where the Higgs sector is located at the Standard Model brane, allowing the matter sector to be distributed between bulk and Standard Model brane in various ways. The resulting 256 possibilities form our 'micro-landscape'. Of course, using our formulae, one could also consider models which, for instance, have Higgs fields in the bulk instead of on the brane and/or have additional vector-like matter. Similarly, $SO(10)$ or other unified gauge groups could be studied. Our analysis is clearly incomplete from this perspective. Nevertheless, restricting ourselves to the simple class of GUT models defined above, we find a minimum in the effective potential at large MR for many of the possible matter distributions. The position of the minimum determines the 4d gauge coupling at the compactification scale. For several of the models, we find results which are in agreement with the phenomenological value of $\alpha_{\mathrm{GUT}} \simeq 1/25$. Thus, two-loop Casimir stabilization offers a simple 'explanation' (within our modest realization of the landscape paradigm) for the appearance of a relatively large 5th dimension and a correspondingly small gauge coupling at the GUT scale.

We have finally discussed two mechanisms for uplifting our AdS_4 vacua without affecting the underlying stabilization mechanism: One possibility is to allow for a small warping, i.e. a small negative 5d cosmological constant. In combination with a brane localized constant superpotential, this induces the required positive contribution to the scalar potential. Alternatively, one can include an extra SUSY-breaking sector on one of the branes (independently of the radion, which is our dominant source of SUSY breaking). Such a brane-localized F term can provide the required uplift. It would be important to find a string-theoretic realization of these uplifting mechanisms and, more generally, to work out in more detail to which extent our stabilization proposal is consistent with a full-fledged underlying heterotic string construction.

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Appendix A

We find that some of our results are not consistent with Ref. [29] where the one-loop mass shifts of the KK-modes due to brane-localized effective operators in a 5d Universal-Extra-Dimensions scenario are calculated. One should be able to obtain our formulae for $c^{(\mathrm{br})}$ by inserting the shifted KK masses which are calculated in Ref. [29] (they discuss only unbroken gauge symmetry however) into Eq. (13). We believe there is an error in the result for the mass corrections of fermions due to scalars in the loop (see Eq. (39) and (B5) in Ref. [29]). In those equations, the number '3' which appears twice should each time be replaced by a '-1'. As a side remark, this implies that the KK masses of the third generation left-handed quark doublet and right-handed top receive a positive contribution due to the Yukawa coupling instead of a negative one (cf. Eq. (45) in Ref. [29]).

We briefly outline the computation we performed as a check (following [53]). Consider the Yukawa theory given by the action[16]

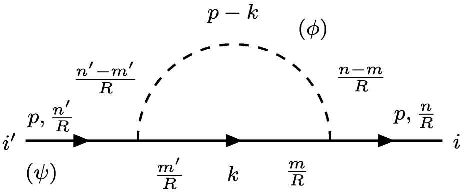
$$\int d^4x \int_{-\pi R}^{\pi R} dy \sqrt{-\det(g_{MN})}\left(\frac{1}{2}M_{P,5}^3\mathcal{R}_5 + \frac{1}{2}\partial_M\phi\partial^M\phi + i\bar{\psi}\Gamma^M\partial_M\psi - h\bar{\psi}\psi\phi\right). \quad (\mathrm{A.1})$$

Consistency requires the parities to be $\phi(-y) = -\phi(y)$ and $\psi(-y) = Z_\psi\gamma_5\psi(y)$ where $Z_\psi \in \{\pm 1\}$. The propagators for the 4d KK modes are

$$\begin{aligned}
\left\langle\phi^{(n)}(p)\phi^{(n')}(p)\right\rangle &= \frac{i}{2}\frac{1}{p^2 - (\frac{n}{R})^2}(\delta_{n,n'} - \delta_{n,-n'}),\\
\left\langle\psi^{(n)}(p)\bar{\psi}^{(n')}(p)\right\rangle &= \frac{i}{2}\frac{\not{p} + i\gamma_5\frac{n}{R}}{p^2 - (\frac{n}{R})^2}(\delta_{n,n'} - Z_\psi\gamma_5\delta_{n,-n'}).
\end{aligned} \quad (\mathrm{A.2})$$

The fermion selfenergy (times $-i$) due to a scalar in the loop then is

[16] Our conventions for the 5d gamma-matrices are $\Gamma^M \equiv (\gamma^\mu, i\gamma^5)$ where the γ^μ are generators of the 4d Clifford algebra and $\gamma^5 \equiv \gamma_5 \equiv i\gamma^0\gamma^1\gamma^2\gamma^3$.



$$= \frac{h^2}{8\pi R}\sum_m I(\not{p},m)(\underbrace{\delta_{n,n'}+Z_\psi\delta_{n,-n'}\gamma_5}_{\to\text{bulk}}\underbrace{-\delta_{2m,(n+n')}-Z_\psi\delta_{2m,(n-n')}\gamma_5}_{\to\text{boundary}}) \tag{A.3}$$

where

$$I(\not{p},m) \equiv \mu^{2\epsilon}\int\frac{d^{4-2\epsilon}k}{(2\pi)^{4-2\epsilon}}\frac{\not{k}+i\gamma_5\frac{m}{R}}{(k^2-(\frac{m}{R})^2)[(p-k)^2-(\frac{n-m}{R})^2]}$$
$$\longrightarrow -i\frac{1}{16\pi^2}\ln\frac{\Lambda^2}{\mu^2}\left(\frac{\not{p}}{2}+i\gamma_5\frac{m}{R}\right) \tag{A.4}$$

and the arrow means taking the divergent part with cutoff Λ and letting $\epsilon \to 0$. We can then write the selfenergy contribution due to the boundary as

$$\sum_m\left(a_1\frac{\not{p}}{2}+a_2 i\gamma_5\frac{m}{R}\right)(\delta_{2m,(n+n')}+Z_\psi\gamma_5\delta_{2m,(n-n')}) \tag{A.5}$$

where

$$a_1 = -\frac{1}{64\pi^2}\frac{h^2}{2\pi R}\ln\frac{\Lambda^2}{\mu^2} = a_2. \tag{A.6}$$

The effective Lagrangian which follows from Eq. (A.5) by Fourier transformation is

$$\delta\mathcal{L}_5 = \frac{\delta(y)+\delta(y-\pi R)}{2}(2\pi R)\left\{a_1 i\bar{\psi}_+\not{\partial}\psi_+ + Z_\psi a_2\left[(\partial_5\bar{\psi}_-)\psi_+ + \bar{\psi}_+(\partial_5\psi_-)\right]\right\} \tag{A.7}$$

where $\psi_\pm = \frac{1}{2}(1 \pm Z_\psi\gamma_5)\psi$. Expanding this into KK modes and comparing it to the tree-level Lagrangian (A.1), one finds that the nth KK mode receives a mass shift

$$\delta m_n = m_n(2a_2 - a_1) = -m_n\frac{1}{64\pi^2}\frac{h^2}{2\pi R}\ln\frac{\Lambda^2}{\mu^2}. \tag{A.8}$$

This disagrees with Eq. (39) in Ref. [29]. Note that, apart from this direct calculation, one also sees that there is an inconsistency among Eqs. (38) and (39) in Ref. [29] because, applied to a supersymmetric theory, they imply that KK gauge bosons and KK gauginos would not have equal masses.

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